

## SOLVING QUADRATIC EQUATIONS

### LEARNING GOALS

- Learn how to solve quadratic equations.

### EQUATIONS

A quadratic equation

- an equation in the form  $ax^2 + bx + c = 0$

-  $a, b, c$  are real numbers with  $a \neq 0$

Solving a quadratic equation

↳ not equal

- means to find the values of the variable that makes the statement true.

- Finding the roots or zeros.

#### EXAMPLE 1: SOLVE BY FACTORING

Solve for  $x$  and check your solution using substitution.

a)  $x^2 + 9x + 14 = 0$

$$\underline{2} \times \underline{7} = 14$$

$$\underline{2} + \underline{7} = 9$$

$$(x+2)(x+7) = 0$$

$$(x-r)(x-s) = 0$$

Zeros  $\Rightarrow -2, -7$

$$x+2=0$$

$$x = -2$$

or

$$x+7=0$$

$$x = -7$$

check

$$(-2)^2 + 9(-2) + 14 = 0$$

$$4 - 18 + 14 = 0$$

$$0 = 0 \quad \checkmark$$

$$(-7)^2 + 9(-7) + 14 = 0$$

$$49 - 63 + 14 = 0 \quad \checkmark$$

b)  $2x^2 + 5x = 0$

$$x(2x+5) = 0$$

$$x = 0$$

or

$$2x+5=0$$

$$x = -\frac{5}{2}$$

$$2(0)^2 + 5(0) = 0$$

$$0 = 0 \quad \checkmark$$

$$2\left(-\frac{5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) = 0$$

$$2\left(\frac{25}{4}\right) + 5\left(-\frac{5}{2}\right) = 0$$

$$\frac{25}{2} - \frac{25}{2} = 0$$

$$0 = 0 \quad \checkmark$$

c)  $6x^2 - x = 15$

$$6x^2 - x - 15 = 0$$

$$\underline{-10} \times \underline{9} = -90$$

$$\underline{-10} + \underline{9} = -1$$

$$\underline{6x^2 - 10x} + \underline{9x - 15}$$

$$2x(3x-5) + 3(3x-5)$$

$$(3x-5)(2x+3) = 0$$

↓  
0

$$x = \frac{5}{3}$$

↓  
0

$$x = -\frac{3}{2}$$

## EXAMPLE 2: PATH OF A STONE

The Path of a stone thrown into a ravine is modeled by the quadratic relation  $y = -x^2 + 5x + 84$ , where  $x$  represents the distance, in metres, travelled horizontally and  $y$  represents the height, in meters, above the surface of the river at the bottom of the ravine. How far does the stone travel horizontally before it hits the water?

$$y = -x^2 + 5x + 84$$

$$-x^2 + 5x + 84 = 0$$

$$\frac{-1(x^2 - 5x - 84)}{-1} = \frac{0}{-1}$$

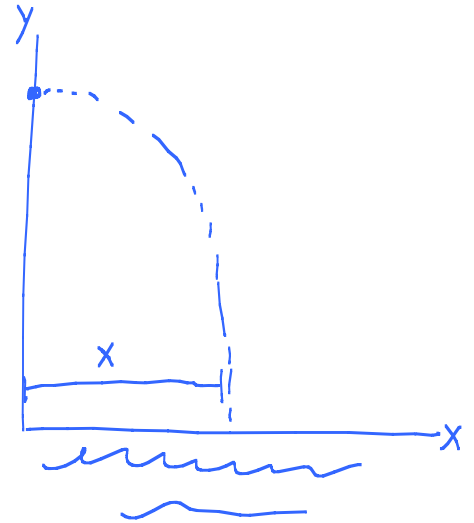
$$x^2 - 5x - 84 = 0$$

$$\frac{-12}{-12} \times \frac{7}{7} = -84$$

$$\frac{-12}{-12} + \frac{7}{7} = -5$$

$$(x - 12)(x + 7) = 0$$

$$\boxed{x = 12} \quad x = -7$$



$\therefore$  the stone travels 12 m before hitting the water.

## EXAMPLE 3: DIMENSIONS OF A RECTANGLE

A rectangle has dimensions  $3x+1$  and  $2x-5$ . Its area is  $1150 \text{ cm}^2$ .  
What are its dimensions?

$$A = lw$$

$$1150 = (3x+1)(2x-5)$$

$$1150 = 6x^2 - 15x + 2x - 5$$

$$1150 = 6x^2 - 13x - 5$$

$$6x^2 - 13x - 5 - 1150 = 0$$

$$6x^2 - 13x - 1155 = 0$$

$$\underline{-90} \times \underline{77} = 6(-1155)$$

$$\underline{-90} + \underline{77} = -13$$

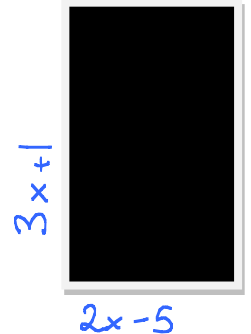
$$\underline{6x^2 - 90x} + \underline{77x - 1155} = 0$$

$$6x(x-15) + 77(x-15) = 0$$

$$(x-15)(6x+77) = 0$$

$$\boxed{x=15} \quad x = \frac{-77}{6}$$

$\therefore$  the dimensions are  
25 cm and 46 cm.



$$\begin{aligned} 2x-5 \\ = 2(15)-5 \\ = 25 \end{aligned}$$

$$\begin{aligned} 3x+1 \\ = 3(15)+1 \\ = 46 \end{aligned}$$

$$l=25 \quad w=46$$

$$\begin{aligned} A &= lw \\ &= 25 \times 46 \\ &= 1150 \text{ cm}^2 \quad \checkmark \end{aligned}$$